

Lesson 10.1: Day 3: Are some groups underrepresented?

According to phys.org, Black and Hispanic females are underrepresented in STEM programs compared to non-STEM programs. A certain university would like to see if this is true for their student population. They took a random sample of 300 STEM students and found that 12 were Black or Hispanic females. A separate random sample of 500 non-STEM students had 75 Black or Hispanic females.

Do the data provide convincing evidence that Black and Hispanic females are underrepresented in STEM programs? Use a 5% significance level.

STATE: Parameter: $P_1 - P_2 \rightarrow$ true difference in proportions of Black and Hispanic females in stem and non stem
 Statistic: $\hat{P}_1 - \hat{P}_2 = .04 - .15 = -.11$

Hypotheses: $P_1 - P_2 = 0$
 $P_1 - P_2 < 0$
 Significance level: 5%

PLAN: Name of procedure: Two sample z test for $P_1 - P_2$

Check conditions:

Random:
 "Random sample of 300"
 "Random sample of 500"

10%:
 $300 < \frac{1}{10}$ STEM students
 $500 < \frac{1}{10}$ non stem students

Large Counts
 $300 \times .04 = 12$
 $300 \times .96 = 288 \geq 10$
 $500 \times .15 = 75$
 $500 \times .85 = 425$

DO: Mean: $\mu_{\hat{P}_1 - \hat{P}_2} = 0$

Standard deviation:

General Formula:
 Test Stat = $\frac{\text{Stat-Null}}{\text{SD}}$

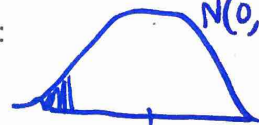
~~$\sigma_{\hat{P}_1 - \hat{P}_2} = \sqrt{\frac{.04 \times .96}{300} + \frac{.15 \times .85}{500}}$~~

Specific Formula:
 $Z = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{\hat{P}_c(1-\hat{P}_c)}{n_1} + \frac{\hat{P}_c(1-\hat{P}_c)}{n_2}}}$

Since $P_1 - P_2 = 0$ we must use $\hat{P}_c = \frac{12 + 75}{300 + 500} = \frac{87}{800} = .109$

Work:

Picture:



$Z = \frac{(.04 - .15) - 0}{\sqrt{\frac{.109(1-.109)}{300} + \frac{.109(1-.109)}{500}}} = -4.838 \approx -4.84$
 Test statistic: -4.84

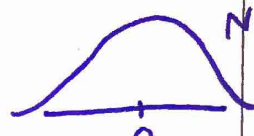
CONCLUDE:

P-value:

.0000006

Assuming there is no difference in proportions of Black and Hispanic females in stem & nonstem, there is a .0000006 probability of getting a difference in sample proportions of -.11 or farther purely by chance. We reject the null. This is statistically significant. We conclude the alternative.

Lesson 10.1: Day 3: A Significance Test for $p_1 - p_2$

Important ideas:	\hat{p}_c = Both proportions combined	
Hypotheses:	$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2 = 0$	
$H_0: p_1 - p_2 = 0$		
$H_a: p_1 - p_2 \neq 0$		
	$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_c(1-\hat{p}_c)}{n_1} + \frac{\hat{p}_c(1-\hat{p}_c)}{n_2}}$	

Check Your Understanding

To study the long-term effects of preschool programs for poor children, researchers designed an experiment. They recruited 123 children who had never attended preschool from low-income families in Michigan. Researchers randomly assigned 62 of the children to attend preschool (paid for by the study budget) and the other 61 to serve as a control group who would not go to preschool. One response variable of interest was the need for social services as adults. Over a 10-year period, 38 children in the preschool group and 49 in the control group have needed social services.

- Do these data provide convincing evidence that preschool reduces the later need for social services for children like the ones in this study? Justify your answer.

State:

$p_1 - p_2 \rightarrow$ true difference in prop. of preschool-control who need S.S.

$H_0: p_1 - p_2 = 0$

$H_a: p_1 - p_2 < 0$

$\hat{p}_1 - \hat{p}_2 = -.19$

$\alpha = .05$
 $\hat{p}_c = \frac{87}{123} = .71$

Plan:

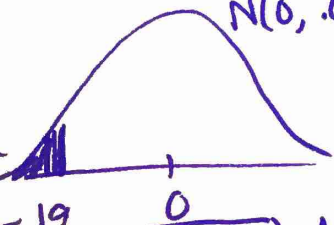
2 sample z test for $p_1 - p_2$

Conditions met.

Do:

Teststat = $\frac{\text{Stat} - \text{Null}}{\text{SD}}$

$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{.71 \cdot .29}{62} + \frac{.71 \cdot .29}{61}}$



$$= \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sigma_{\hat{p}_1 - \hat{p}_2}} = \frac{-0.19 - 0}{0.082} = -2.32 \rightarrow \boxed{.0102}$$

Conclude:

Assuming there is no difference in proportions there is a .0102 prob. of getting $\hat{p}_1 - \hat{p}_2 = -.19$ or less purely by chance.

- Based on your conclusion to Question 1, could you have made a Type I error or a Type II error? Explain your reasoning.

Type I error because we rejected the null.

Reject the null.