Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

Rational Zeros

- Recall that a polynomial function of degree n can have at most $\underline{n}$ real zeros.
- Real zeros can be $\underline{\text{Positive}}$ or $\underline{\text{negative}}$.
- Rational zeros are those that can be written in the form of a $\underline{p/q}$.

Rational Zero Theorem

The **Rational Zero Theorem** can be used to determine all possible *rational* zeros of a polynomial function.

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ has integer coefficients, then every rational zero of the function has the following form:

$$\boxed{\frac{p}{q} = \frac{\pm \text{ factors of constant}}{\pm \text{ factors of leading coefficient}}}$$

List Possible Rational Zeros

put every p over every q

Directions: List all possible rational zeros of each function.

1. $f(x) = x^3 + 2x^2 - x + 2$

p = 2
q = 1

$p = \pm 1, \pm 2$
$q = \pm 1$

$\boxed{\pm 1, \pm 2}$

2. $f(x) = x^5 + 9x^3 - 2x^2 - 18$

p = 18
q = 1

$p = \pm 18, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1$
$q = \pm 1$

$\boxed{\pm 18, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1}$

3. $f(x) = 4x^3 - 4x^2 - x + 1$

p = 1
q = 4

$\dfrac{p}{q} = \dfrac{\pm 1}{\pm 4, \pm 2, \pm 1}$

$\boxed{\pm \frac{1}{4}, \pm \frac{1}{2}, \pm 1}$

4. $f(x) = 3x^3 - x^2 - 18x + 16$

p = 16
q = 3

$p = \pm 16, \pm 8, \pm 4, \pm 2, \pm 1$
$q = \pm 3$

$\boxed{\pm \frac{16}{3}, \pm \frac{8}{3}, \pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}}$

5. $f(x) = 6x^5 - 8x^3 + 9x^2 - 12$

p = 12
q = 6

$\dfrac{p}{q} = \dfrac{\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1}{\pm 6, \pm 3, \pm 2, \pm 1}$

See other key

6. $f(x) = 2x^3 + 11x^2 + 28x + 24$

p = 24
q = 2

$p = \pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$
$q = \pm 2, \pm 1$

see other key

$\pm\frac{12}{6} \to \pm 2$; $\pm\frac{6}{6} \to \pm 1$; $\pm\frac{4}{6} \to \pm\frac{2}{3}$; $\pm\frac{3}{6} \to \pm\frac{1}{2}$; $\pm\frac{2}{6} \to \pm\frac{1}{3}$; $\pm\frac{1}{6}$

$\pm\frac{12}{3} \to \pm 4$; $\pm\frac{6}{3} \to \pm 2$; $\pm\frac{4}{3}$; $\pm\frac{3}{3} \to \pm 1$; $\pm\frac{2}{3}$; $\pm\frac{1}{3}$

$\pm\frac{12}{2} \to \pm 6$; $\pm\frac{6}{2} \to \pm 3$; $\pm\frac{4}{2} \to \pm 2$; $\pm\frac{3}{2}$; $\pm\frac{2}{2} \to \pm 1$; $\pm\frac{1}{2}$

$\pm\frac{12}{1} \to \pm 12$; $\pm\frac{6}{1} \to \pm 6$; $\pm\frac{4}{1} \to \pm 4$; $\pm\frac{3}{1} \to \pm 3$; $\pm\frac{2}{1} \to \pm 2$; $\pm\frac{1}{1} \to \pm 1$

I put every P over every Q, and then reduced. Now I will put them in order and eliminate duplicates.

$\boxed{\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 3/2, \pm 4/3, \pm 1, \pm 2/3, \pm 1/2, \pm 1/3, \pm 1/6}$

#5

$\pm\frac{24}{2} = \pm 12$; $\pm\frac{12}{2} = \pm 6$; $\pm\frac{8}{2} = \pm 4$; $\pm\frac{6}{2} = \pm 3$; $\pm\frac{4}{2} = \pm 2$; $\pm\frac{3}{2} = \pm\frac{3}{2}$; $\pm\frac{2}{2} = \pm 1$; $\pm\frac{1}{2}$

$\pm\frac{24}{1} = \pm 24$; $\pm\frac{12}{1} = \pm 12$; $\pm\frac{8}{1} = \pm 8$; $\pm\frac{6}{1} = \pm 6$; $\pm\frac{4}{1} = \pm 4$; $\pm\frac{3}{1} = \pm 3$; $\pm\frac{2}{1} = \pm 2$; $\pm\frac{1}{1} = \pm 1$

#6

$\boxed{\pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 3/2, \pm 1, \pm 1/2}$

* This tells us that there are 20 possible rational solutions. Later on we will work to eliminate the wrong ones.

#7) $f(x) = x^3 + 3x^2 - 6x - 8$

Find possible solutions

$$\frac{p}{q} \quad \frac{\pm 8, \pm 4, \pm 2, \pm 1}{\pm 1}$$

so, $\frac{\pm 8}{1}, \frac{\pm 4}{1}, \frac{\pm 2}{1}, \frac{\pm 1}{1}$

$\boxed{\pm 8, \pm 4, \pm 2, \pm 1}$

* remember this means possible *
solutions are

$+8, -8, +4, -4, +2, -2, +1, -1$

Now we use synthetic substitution to find ones that work.

$$\begin{array}{r|rrrr} 8 & 1 & 3 & -6 & -8 \\ & \downarrow & 8 & 88 & 656 \\ \hline & 1 & 11 & 82 & 648 \end{array}$$

8 does not work

$$\begin{array}{r|rrrr} -8 & 1 & 3 & -6 & -8 \\ & \downarrow & -8 & -40 & 368 \\ \hline & 1 & 5 & -46 & 360 \end{array}$$

-8 does not work

$$\begin{array}{r|rrrr} 4 & 1 & 3 & -6 & -8 \\ & \downarrow & 4 & 28 & 88 \\ \hline & 1 & 7 & 22 & 80 \end{array}$$

4 does not work

$$\begin{array}{r|rrrr} -4 & 1 & 3 & -6 & -8 \\ & \downarrow & -4 & 4 & 8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$\boxed{-4 \text{ works}}$

$x^2 - x - 2$

Use what is left

$x^2 - x - 2 = 0$

$$\begin{array}{c|c} x & x \\ \hline -2 & +1 \end{array} \qquad -2, \ -1$$

$(x-2)(x+1) = 0$

$x - 2 = 0 \qquad x + 1 = 0$

$\boxed{x = 2} \qquad \boxed{x = -1}$

The three solutions are $x = -4, 2, -1$