The Binomial Distribution

6.3

4 commands of binomial distributions!

- Each trial results in a success or failure
- There is a fixed number (n) of trials
- The trials are independent (knowing result of one trial does not affect the other trials)
- The probability (p) of success is the same for each trial

Two outcomes of interest

- We use a coin toss to see which of the two football teams gets the choice of kicking off or receiving to begin the game.
- A basketball player shoots a free throw; the outcomes of interest are {she makes the shot; she misses}
- A young couple prepares for their first child; the possible outcomes are {boy; girl}
- A quality control inspector selects a widget coming off the assembly line; he is interested in whether or not the widget meets production requirements.

Terms

- If data are produced in a binomial setting, then the random variable X = number of successes is called a ***binomial random variable***, and the probability distribution of X is called a *binomial distribution*.
- There are times when it IS sufficient for outcomes of an event to be “close enough” to independent to use binomial rules.

Binomial Distribution

- The distribution of the count X of successes in the binomial setting is the binomial distribution with parameters n and p.
- The parameter n is the number of observations or trials, and p is the probability of a success on any one observation or trial.
- The possible values of X are the whole numbers from 0 to n.
- As an abbreviation, we say that X is B(n, p).

Example 1

- Blood type is inherited. If both parents carry genes for the O and A blood types, each child has probability 0.25 of getting two O genes and so of having blood type O. Different children inherit independently of each other. The number of O blood types among 5 children of these parents is the count X of successes in 5 independent observations with probability 0.25 of a success on each observation. So X as the binomial distribution with $n = 5$ and $p = .25$. We say that X is $B(5, 0.25)$

Example 2

- Deal 10 cards from a shuffled deck and count the number X of red cards. There are 10 observations, and each gives either a red or a black card. A “success” is a red card. But the observations are not independent. If the first card is black, the second is more likely to be red because there are more red cards than black cards left in the deck. The count X does not have a binomial distribution.

Example 3

- An engineer chooses an SRS of 10 switches from a shipment of 10,000 switches. Suppose that (unknown to the engineer) 10% of the switches in the shipment are bad. The engineer counts the number X of bad switches in the sample. This is not quite a binomial setting. Removing one switch changes the proportion of bad switches remaining in the shipment. So the state of the second switch chosen is not independent of the first. But removing one switch from a shipment of 10,000 changes the makeup of the remaining 9999 switches very little. In practice, the distribution of X is very close to the binomial distribution with $n = 10$ and $p = .1$.

FORMULAS

$$\mu_X = np$$

$$\sigma_X = \sqrt{np\left(1 - p\right)}$$

- YES! GIVEN ON AP EXAM!

P.D.F.

- Given a discrete random variable X, the ***probability distribution function*** assigns a probability to each value of X. The probabilities must satisfy the rules for probabilities given in Chapter 6.
- The command binompdf(n,p,X) calculates the binomial probability of the value X. It is found under 2^{nd}/DISTR/0:binompdf

Back to Example on blood

- The command binompdf(5, .25, 3) calculates the binomial probability that X = 3 to be .08789

- This means the probability of 3 of the children will be type O blood is 8.8%

C.D.F.

- Given a random variable, X, the ***cumulative probability distribution*** of X calculates the sum of the probabilities for 0, 1, 2, …. Up to the value of X.
- It calculates the probability of obtaining at most X successes in *n* trials.

Back to blood types…

- To determine the probability of **at most 3** children having type O blood:
- Binomcdf(5,.25,3) = .984
- Interpretation: about 98.4% probability that 0,1,2,3 children will have type O blood.

Calculator summary!

- P(X = x)

  use binompdf (n, p, x) where n = # of observations, p is the probability and x is the value you are identifying

  P(X$\leq$x)

  binomcdf(n,p,x)

  P(X>x)

  1-binomialcdf (n,p,x)

- P(X <x)

  Binomcdf (n,p, x-1)

Binomial Example

- A baseball pitcher throws 30 pitches in an inning. The pitcher throws a strike 60% of the time. Is this setting binomial?
  - 1. success or failure?
  - 2. fixed number of observations?
  - 3. independent observations?
  - 4. is the probability the same for each observation?

How many strikes does the pitcher expect to throw?

np= (30)(.60) = 18

What is the standard deviation?

$$\sigma_X = \sqrt{np(1-p)}$$

What is the probability that he throws exactly 21 strikes in an inning? P (X=21) = binompdf (30, .6, 21)

Ex. Cont.

- What is the probability that he throws 15 or fewer strikes? $P(X \leq 15) = \text{binomcdf} (30, .6, 15)$

- What is the probability that he throws more than 11 strikes? $P (X > 11) = 1- \text{binomcdf} (30,.6, 11)$

- What about between 12 and 20 strikes?

  $P (12<X<20) =$