The Geometric Distribution
6.3

Consider the following situations:

- Flip a coin until you get a head.
- Roll a die until you get a 3.
- In basketball, attempt a three-point shot until you make a basket.

Notice…

- All of these situations involve counting the number of trials until an event of interest happens.
- The possible values of the number of trials is an infinite set because it is theoretically possible to proceed indefinitely without ever obtaining a success.

Commands of the Geometric!

- Each observation falls into one of just two categories, which for convenience we call “success” or “failure.”
- The probability of a success, *p*, is the same for each observation.
- The observations are all independent.
- The variable of interest, X, is the number of trials required to obtain the first success.

Example

- An experiment consists of rolling a single die. The event of interest is rolling a 3; this event is called a success. The random variable is defined as X = the number of trials until a 3 occurs. Is this a geometric setting?

Example

- Suppose you repeatedly draw cards without replacement from a deck of 52 cards until you draw an ace. There are two categories of interest: ace = success; not ace = failure. Is this a geometric setting?

Rule for Calculating Geometric Probabilities

- If X has a geometric distribution with probability p of success and (1 – p) of failure on each observation, the possible values of X are 1, 2, 3, …. If n is any one of these values, then the probability that the first success occurs on the nth trial is:

$$P(X = n) = (1 - p)^{n-1} p$$

Geometric Probability Distribution

- A probability distribution table for the geometric random variable is strange indeed because it never ends; the number of table entries is infinite.

X	1	2	3	4	5 …
P(X)	p	(1-p)p	$(1-p)^2p$	$(1-p)^3p$	$(1-p)^4p$ …

The mean of a Geometric Random Variable

- If X is a geometric random variable with probability of success p on each trial, then the mean, or expected value, of the random variable, that is, the expected number of trials required to get the first success, is

$$\mu = \frac{1}{p}$$

Probability

- The probability that it takes more than n trials to see the first success is

$$P(X > n) = (1 - p)^n$$

Or 1-geometcdf(p,n)

- The standard deviation is: $\sigma = \sqrt{\frac{1-p}{p^2}}$

Calculator Commands

- Geometpdf (p, X) with p being the probability of success and X being the number of the trial on which the first success occurs.

- Geometcdf (p,X) where X is the maximum number of trials to get the first success.

Examples

- Brady, the pitcher, is starting a new inning. The probability that he'll throw a strike is still .6
- What is the expected number of pitches needed to pitch a strike?

- What is the probability that he will throw his first strike on the 5^{th} pitch?

- What is the probability that it will take more than 5 pitches before he throws his first strike?